Grid generation using lemniscates of two foci

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Abstract

We consider the approximation of planar regions by pairs of lemniscates that are stitched together continuously. Lemniscates are level curves of the absolute value of an univariate complex polynomial. We approximate coastal sides or 2D meander like regions such that profiles of arteries, rivers and sinuous roads using lemniscates of two foci and use confocal lemniscates to build orthogonal grids. The use of low degree polynomials gives some computational advantages which are exploited in the implementation. The main applications are numerical solution of PDE’s, computer aided surgical procedures and computer aided geometric design. We report on the results of numerical experiments performed with MATLAB® using data from the Orinoco River, the San Andrés Island and the Guajira Peninsula. In this paper we compare the exact distance between a point and a lemniscate with the estimate we have used in previous works.

1. Introduction. The construction of grids on irregular regions has been an important topic in the development of software to solve numerically partial differential equations. The discretization, via finite differences say, of a set of partial differential equations on a 2D region requires a mesh defined on the boundary and the interior where the properties of the phenomenon to be studied will be measured. In the literature, the most popular numerical procedure for generating a mesh on 2D regions are: conformal mapping methods, elliptic grid generation and variational methods. More recently the use of lemniscates to construct orthogonal grids has been presented as an competitive alternative see references [1,2]. Here we present the results using lemniscates of 2 foci for meander-like regions and coastlines.

2. Numerical Results. Figures 2 and 3 illustrate the results for a satellite image of the Orinoco river in the Colombia-Venezuela border, see Figure 1. The Orinoco piece exhibited is encased in a twenty by twenty miles square off Puerto Páez and Puerto Carreño. Figure 2 shows nineteen lemniscatic sectors and the corresponding pairs of confocal lemniscates. Figure 3 illustrates the resulting orthogonal mesh.
Aproximación de las riberas del Orinoco por sectores lemniscáticos

Orthogonal mesh for the Orinoco River

3. Conclusions. The approximation of meander-like regions by confocal lemniscates and of costal sides by lemniscates gives the possibility of building good orthogonal meshes.

4. Related works.

